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SIMULATION OF RANDOM WIND FLUCTUATIONS

By Morris Perlmutter  
Northrop Services, Inc.  
Huntsville, Alabama

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## **FOREWORD**

This report presents the results of work done by Northrop Services, Inc., Huntsville, Alabama, for the National Aeronautics and Space Administration, George C. Marshall Space Flight Center, under Contract NAS8-21810. The work was performed for the Science and Engineering Directorate in response to Appendix A, Schedule Order Number A022(A-13)

Dr. George H. Fichtl was the Technical Coordinator for this task. The author is grateful to Dr. Fichtl for his guidance and fruitful discussions during this work.

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## **Section I**

### **INTRODUCTION**

Future studies of the ascent control of the Space Shuttle are being planned. Some of these planned studies will be carried out analytically using a model simulation procedure. In this technique the Space Shuttle is followed analytically as it ascends through the atmosphere. The model interacts with the simulated random winds, and the ability of the ascent control to maintain the desired trajectory is studied.

The simulated winds must be generated so as to have the appropriate statistical behavior. This report discusses the method of generating a random wind signal using a digital computer that will have the same statistics as the winds encountered in the Space Shuttle ascent.

The procedure for generating the random wind is to develop a control system which inputs discrete white Gaussian noise and outputs a random signal that has the statistical behavior of the wind. The control system is written in terms of state equations which are then digitized for computer calculations.

## Section II

### SIGNAL ANALYSIS

#### 2.1 EMPIRICAL AUTOCORRELATION

An empirical wind autocorrelation was obtained from detailed Jimsphere measurements (Reference 1) made at the Kennedy Space Center and shown on Figure 2-1. The random components of the longitudinal winds,  $u$  and  $v$ , were obtained as a function of altitude  $z$ . The turbulent wind component was normalized to yield

$$y(t) = \frac{v(t)}{\sigma(t)}$$

where  $t = z/L(z)$ . The term  $L(z)$  is a length scale which is chosen so that the dimensionless  $y$  process is homogeneous; that is, the second order statistics of  $y$  are independent of  $z$ . The  $\sigma(z)$  is the standard deviation of  $v(z)$ .

The resulting empirical autocorrelation is given by

$$R_y(t_1, t_2) = \langle y(t_1) y(t_2) \rangle = \langle y(t_1) y(t_1 + \tau) \rangle = R_y(\tau)$$

where  $\langle \cdot \rangle$  represents an ensemble average and  $\tau$  is the lag. The same empirical autocorrelation was found to apply to both the  $u$  and the  $v$  components of the horizontal wind when they were appropriately normalized (Reference 1).

The empirical autocorrelation can be represented in a functional form which can be Fourier transformed to give the power spectrum for the dimensionless wind. By factoring the power spectrum, a control system function can be obtained. A control system is then defined which inputs white noise and outputs a random signal which has the same autocorrelation as the functional autocorrelation just discussed. The control system can then be written in terms of state equations which are put in a discrete form for use on a digital computer. The autocorrelation of the digitized output signal is in good agreement with the desired autocorrelation.

**Figure 2-1. AUTOCORRELATION OF WIND SIGNAL**

## 2.2 FITTING EMPIRICAL AUTOCORRELATION

An empirical autocorrelation was obtained from wind data (Reference 1) and is plotted on Figure 2-1. The empirical autocorrelation was approximated in functional form by

$$\begin{aligned} R_y(\tau) &= \langle y(t) y(t + \tau) \rangle \\ &= \exp(-D|\tau|) \left\{ \cos B(\tau) - \frac{D}{B} \sin B|\tau| \right\} \end{aligned} \quad (2-2a)$$

where the notation  $\langle \cdot \rangle$  refers to ensemble averages and B and D are empirically determined coefficients.

$$\begin{aligned} B &= 1.122 \text{ }^\circ \\ D &= 0.539 \end{aligned} \quad (2-2b)$$

and  $\tau$  is the autocorrelation lag. A comparison of the empirical and functional form of the autocorrelation in Figure 2-1 shows good agreement between the two.

By taking the Fourier transform of the autocorrelation the power spectrum  $\phi_y$  is obtained; and, since  $R_y$  is an even function, the power spectrum can be written as

$$\phi_y = 2 \int_0^\infty R_y(\tau) \cos \omega \tau d\tau = \frac{4D\omega^2}{[D^2 + (B-\omega)^2][D^2 + (B+\omega)^2]} \quad (2-3)$$

## 2.3 SYSTEM FUNCTION

In general, the output power spectrum of a system can be written as

$$\phi_y = HH^* \phi_I \quad (2-4)$$

where  $\phi_I$  is the input power spectrum and H is the system function. The  $H^*$  is the complex conjugate of the system function.



If the input is white Gaussian noise, then  $\phi_I = 1.0$ ; and  $\phi_y$  equals  $HH^*$  where  $H$  is given by

$$H = \frac{(2\sqrt{D}) S}{(S + D - iB)(S + D + iB)} \quad (2-5)$$

where  $S = i\omega$ . This result is shown by Figure 2-2. The system equation can now be written as

$$Y(S) = H(S) I(S)$$

where  $Y(S)$  is the dimensionless wind having the autocorrelation given by Equation 2-2 and  $I(S)$  is the Gaussian white noise input (Figure 2-3).

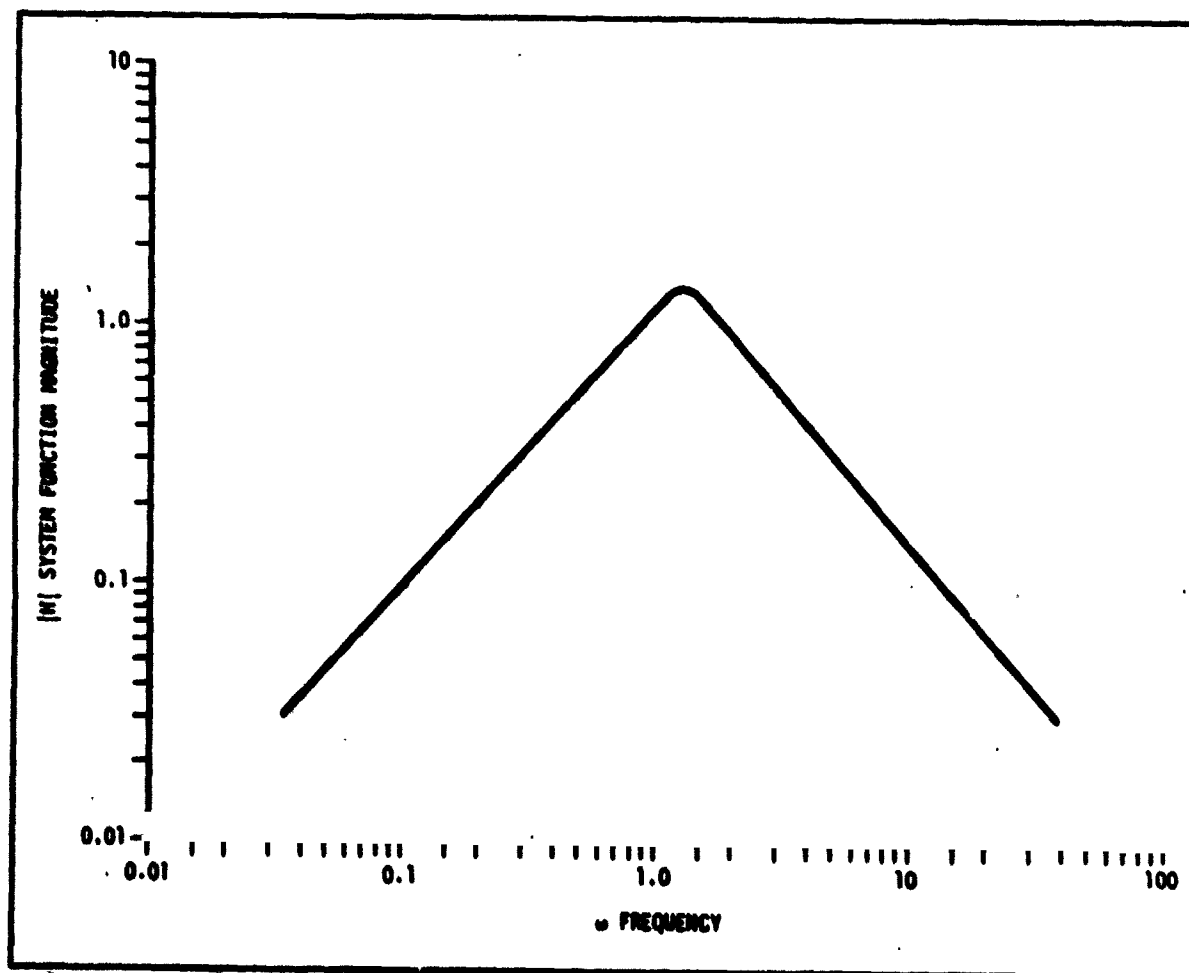


Figure 2-2. MAGNITUDE OF SYSTEM FUNCTION

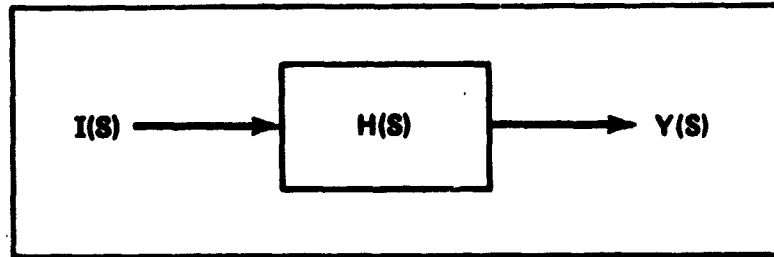


Figure 2-3. CONTROL SYSTEM

## 2.4 STATE SPACE SYSTEM

As shown in Reference 2, we can define the system in terms of state variable  $X_i$  given by the following equation where the Einstein summation convention is implied by repeated indices:

$$\frac{dX_i}{dt} = a_{ij} X_j + d_i I \quad (2-7)$$

The system output is given by

$$Y = e_i X_i \quad (2-8)$$

Following Dorf (Reference 2) the flow graph state model is produced as shown on Figure 2-4, where

$$b_1 = 2\sqrt{D}; \quad a_1 = 2D \text{ and } a_0 = D^2 + B^2 \quad (2-9)$$

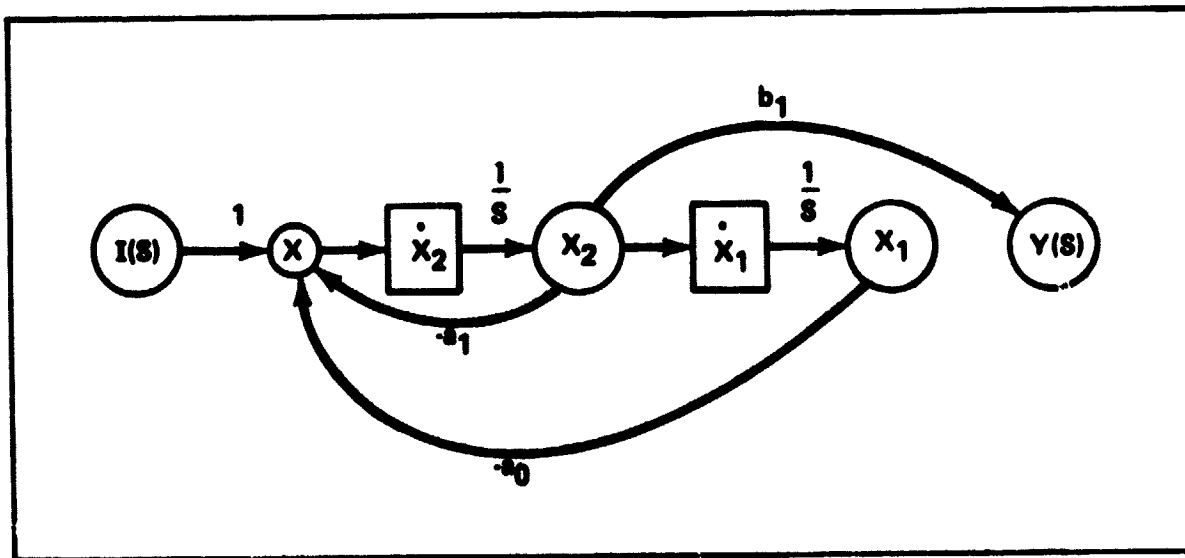


Figure 2-4. FLOW GRAPH STATE MODEL

Then, the matrices can be written as

$$[a_{1j}] = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \quad (2-10)$$

$$[d_1] = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2-11)$$

$$[e_1] = \begin{bmatrix} 0 \\ b_1 \end{bmatrix} \quad (2-12)$$

### Section III DISCRETE TIME SYSTEM

#### 3.1 DISCRETE STATE SPACE SYSTEM

For use on digital computers the state equations must be converted to a discrete time system. One procedure for achieving this is given in Reference 3. In this procedure the input signal  $I$  is passed through a zero order holding device which samples the signal at unit intervals of time and holds the signal value constant between samples (Figure 3-1). The procedure for converting to a discrete time system is discussed next.

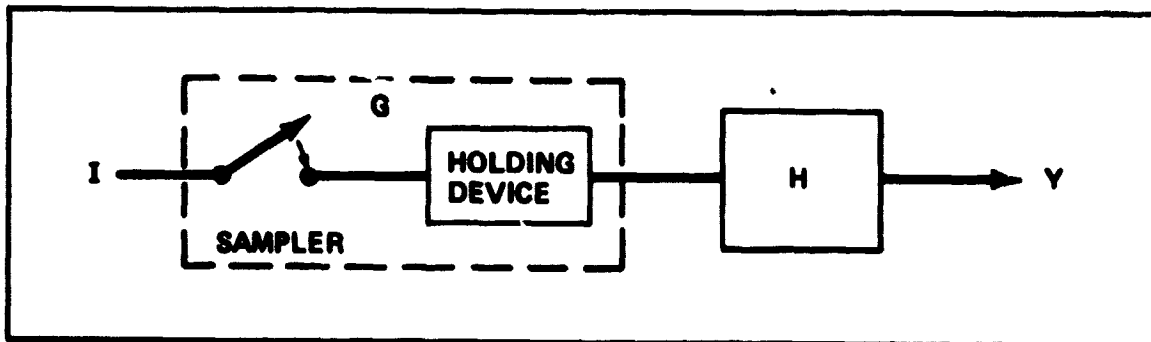


Figure 3-1. CONTROL SYSTEM WITH SAMPLER AND HOLDING DEVICE

Equation 2-7 can be integrated as shown in Reference 4 to give

$$X_1(t) = \phi_{1J}(t - t_0) X_J(t_0) + \int_{t_0}^t \phi_{1J}(t - \tau) d_J I(\tau) d\tau \quad (3-1)$$

where  $\phi_{1J}$  is known as the fundamental matrix. Since  $I$  is considered constant over the interval  $T$ , Equation 3-1 can be evaluated at time  $t = (K + 1)T$  over the increment  $T$  and obtain

$$X_1(K + 1) = \phi_{1J}(T) X_J(K) + A_1(T) I(K) \quad (3-2)$$

where

$$\Lambda_1(T) = \int_0^T \phi_{1J}(\tau) d_J d\tau \quad (3-3)$$

Following the usual procedures, the fundamental matrix can be evaluated using Laplace transforms where  $L$  represents the Laplace transform operation.

$$L[\phi_{1J}(T)] = [S\delta_{1J} - a_{1J}]^{-1} \quad (3-4)$$

where

$$\delta_{1J} = \begin{cases} 0 & \text{if } i \neq J \\ 1 & \text{if } i = J \end{cases}$$

This can be solved to give

$$[\phi_{1J}(T)] = \begin{bmatrix} e^{-DT} \left[ \cos BT + \frac{D}{B} \sin BT \right] & \frac{1}{B} e^{-DT} \sin BT \\ -\frac{a_0}{B} e^{-DT} \sin BT & e^{-DT} \left[ \cos BT - \frac{D}{B} \sin BT \right] \end{bmatrix} \quad (3-5)$$

Taking the limit for small  $T$

$$[\phi_{1J}(T)] = \begin{bmatrix} 1 & T \\ -a_0 T & 1-2DT \end{bmatrix} \approx [\Lambda_{1J}] ; \quad 0 < T \ll 1 \quad (3-6)$$

Then, from Equation 3-3, obtain

$$[\Lambda_1(T)] = \int_0^T \begin{bmatrix} \tau \\ 1-2D\tau \end{bmatrix} d\tau = \begin{bmatrix} 0 \\ T \end{bmatrix} \approx [D_1] ; \quad 0 < T \ll 1 \quad (3-7)$$

This then gives the result for the discrete case as

$$X_1(K+1) = \Lambda_{1J} X_J(K) + D_1 I(K) \quad (3-8)$$

Where  $\Lambda_{1J}$  and  $D_1$  are defined by Equations 3-6 and 3-7.

This same relationship can be obtained by using a forward finite differencing technique (Reference 2). In this method Equation 2-7 can be written as

$$\frac{X_1(K+1) - X_1(K)}{T} = a_{1J} X_J(K) + d_1 I(K) \quad (3-9)$$

This expression can be rewritten to give the same result as Equation 3-8.

$$\begin{aligned} X_1(K+1) &= (Ta_{1J} + \delta_{1J}) X_J(K) + Td_1 I(K) \\ &= A_{1J} X_J(K) + D_1 I(K) \end{aligned} \quad (3-10)$$

### 3.2 EFFECT OF DIGITIZING ON AUTOCORRELATION

As shown on Figure 3-1, a zero order holding device has been added (Reference 4). Therefore, the input into the continuous system will be put in a discrete form consisting of a stair function of Gaussian heights and of width T (Figure 3-1). As discussed in Reference 5 a correction must be made for this effect. This can be done by finding the spectrum of the discrete input. The autocorrelation of the discrete input is given by

$$\begin{aligned} R_I'(\tau) &= \langle I'(t) I'(t + \tau) \rangle \\ &= \sigma^2 \Pr[A] \end{aligned} \quad (3-11)$$

The  $\sigma^2$  is the variance of the Gaussian noise input, and  $\Pr[A]$  is the probability that points  $t$  and  $t + \tau$  of the input in the discrete form both occur between the times  $KT$  and  $(K+1)T$ . Since the Gaussian noise input has a variance of 1.0 then  $\sigma^2 = 1$ . The probability of  $A$  occurring (Reference 6) can be seen to be

$$\Pr(A) = \begin{cases} \frac{T - |\tau|}{T}, & |\tau| \leq T \\ 0, & |\tau| > T \end{cases} \quad (3-12)$$

Then the input autocorrelation is given by

$$R_I'(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & |\tau| \leq T \\ 0, & |\tau| > T \end{cases} \quad (3-13)$$

This can be Fourier transformed to give the input power spectrum

$$\phi_I' = T \left[ \frac{\sin(\omega T/2)}{\frac{T\omega}{2}} \right]^2 \sim T ; 0 < T \ll 1 \quad (3-14)$$

This same result was found in Reference 5.

Thus, the output spectrum of the discrete input,  $y'$ , is now given by

$$\phi_{y'} = THH^* \phi_I = T \phi_y \quad (3-15)$$

Therefore, the digitized autocorrelation is

$$R_{y'}(\tau) = TR_y(\tau) \quad (3-16)$$

Thus, the output signal must be normalized to obtain the correct autocorrelation

$$\text{by } \left[ R_{y'}(0) \right]^{1/2} = \sigma_{y'} = T^{1/2}$$

## Section IV

### THEORETICAL DISCRETE AUTOCORRELATIONS

The theoretical autocorrelation can be calculated for the discrete equation following the procedure in Reference 5 by writing Equation 3-8 as

$$\begin{aligned} X_1(K+1) &= A_{1J} X_J(K) + D_J I(K) \\ X_1(K+2) &= A_{1J} X_J(K+1) + D_J I(K+1) \\ &= A_{1K}^{(2)} X_K(K) + A_{1K} D_K I(K) + D_1 I(K+1) \end{aligned} \quad (4-1)$$

where

$$A_{1K}^{(2)} = A_{1J} A_{JK} \quad (4-2)$$

Similarly

$$\begin{aligned} X_1(K+3) &= A_{1K}^{(3)} X_K(K) + A_{1K}^{(2)} D_K I(K) + A_{1K} D_K I(K+1) \\ &\quad + D_1 I(K+2) \end{aligned} \quad (4-3)$$

and, in general,

$$X_1(K+n) = A_{1m}^{(n)} X_m(K) + \sum_{r=K}^{K+n-1} A_{1J}^{(K+n-1-r)} D_J I(r) \quad (4-4)$$

where

$$A_{1J}^{(0)} D_J = D_1 \quad (4-5)$$

Letting  $K+n = l$ , and, as  $K \rightarrow -\infty$ ,  $X_m(-\infty) = 0$

then

$$X_1(l) = \sum_{r=-\infty}^{l-1} A_{1J}^{(l-1-r)} D_J I(r) \quad (4-6)$$



This gives a nonrecursive form for obtaining  $X_1(l)$ . If  $m = l-1-r$  then the equation becomes

$$X_1(l) = \sum_{m=0}^{m=\infty} A_{1J}^{(m)} D_J I(l-m-1) \quad (4-7)$$

Since  $\langle I \rangle = 0$  then  $\langle X_1 \rangle = 0$

The autocorrelation can be found as follows:  
using Equation 4-7

$$\langle I(l) X_1(l+\lambda) \rangle = R'_{IX_1}(\lambda) = \sum_{m=0}^{m=\infty} A_{1J}^{(m)} D_J \langle I(l) I(l+\lambda-m-1) \rangle \quad (4-8)$$

Since  $I$  is Gaussian white noise

$$\langle I(l) I(l+\lambda-m-1) \rangle = \delta(\lambda-m-1) ; \quad \text{where } \delta = \begin{cases} 0 & \text{when } \lambda-m-1 \neq 0 \\ 1 & \text{when } \lambda-m-1 = 0 \end{cases} \quad (4-9)$$

So that Equation 4-8 becomes

$$R'_{IX_1}(\lambda) = \begin{cases} A_{1J}^{(\lambda-1)} D_J & \text{for } \lambda \neq 0 \\ 0 & \text{for } \lambda = 0 \end{cases} \quad (4-10)$$

Similarly,

$$\begin{aligned} R'_{X_1 X_K}(\lambda) &= \langle X_1(l) X_K(l+\lambda) \rangle = \sum_{m=0}^{\infty} A_{1J}^{(m)} D_J \langle I(l-m-1) X_K(l+\lambda) \rangle \\ &= \sum_{m=0}^{\infty} A_{1J}^{(m)} D_J R'_{IX_K}(\lambda+m+1) \end{aligned} \quad (4-11)$$

or

$$R'_{X_1 X_K}(\lambda) = \sum_{m=0}^{\infty} A_{1J}^{(m)} D_J A_{Kl}^{(\lambda+m)} D_l = \sum_{m=0}^{\infty} f(m) f(m+\lambda) \quad (4-12)$$

The result can be seen to be a convolution summation which is the discrete time counterpart to the convolution integral.

For the autocorrelation of the system output y

$$R_y(\lambda) = \frac{\langle y(l) y(l+\lambda) \rangle}{\sigma_y'^2} = \frac{b_1^2 \langle x_2(l) x_2(l+\lambda) \rangle}{\sigma_y'^2} = \frac{b_1^2 R_{x_2 x_2}(\lambda)}{\sigma_y'^2} \quad (4-13)$$

Then, by combining Equations 4-12 and 4-10, obtain

$$R_y(\lambda) = \frac{b_1^2 T^2 \sum_{m=0}^{\infty} A_{22}^{(m)} A_{22}^{(m+\lambda)}}{\sigma_y'^2} = \frac{\sum_{m=0}^{\infty} A_{22}^{(m)} A_{22}^{(m+\lambda)}}{\sum_{m=0}^{\infty} (A_{22}^{(m)})^2} \quad (4-14)$$

where the normalizing factor is given by

$$\sigma_y'^2 = \langle y^2(l) \rangle = b_1^2 R_{x_2 x_2}(0) = b_1^2 T^2 \sum_{m=0}^{\infty} (A_{22}^{(m)})^2 \quad (4-15)$$

The analytical result for the discrete autocorrelation  $R_y(\lambda)$  is shown in Figure 2-1 for  $T = 0.125$  and  $T = 0.1$  and is in good agreement with the desired autocorrelation. The values found for  $\sigma_y'^2$  were 0.145 for  $T$  of 0.125. For  $T = 0.1$ ,  $\sigma_y'^2 = 0.111$ . This indicates that the system is stable since the variance is finite. The correction factor found earlier in Equation 3-16 was  $\sigma_y'^2 = T$ , which in the present case gives close agreement with the more detailed discrete results.

## Section V

### STABILITY ANALYSIS

In Reference 4 it is shown that a stationary linear system subjected to a bounded input is stable if and only if all the zeros of the characteristic polynomial,  $|\lambda I_{\delta J} - A_{1J}|$  be within the circle  $|\lambda| = 1$  in the complex  $\lambda$  plain. This results in

$$|\lambda I_{\delta J} - A_{1J}| = 0 \quad (5-1)$$

where  $|\cdot|$  denotes a determinant. The eq. 5-1 can be expanded to

$$\lambda^2 + \lambda(2DT - 2) + 1 - 2DT + a_0 T^2 = 0 \quad (5-2)$$

This can be solved to give

$$\lambda = (1 - DT) \pm iTB \quad (5-3)$$

which has the magnitude

$$|\lambda| = \left[ 1 - 2DT + T^2(D^2 + B^2) \right]^{1/2} \quad (5-4)$$

Since  $(D^2 + B^2)T < -2D$  for small  $T$  then  $|\lambda| < 1$  ; and therefore, the system is stable.

## Section VI

### COMPUTER SIGNAL OUTPUT

Discrete Gaussian white noise can be generated on the computer using readily available programs. Inputting this into the recursive equation for  $X_1$  (Equation 3-8) can result in a set of discrete values of  $y(K)$ ;  $K = 1, 2, 3, \dots$ . This result can be normalized either by  $\sigma_y'$  (Equation 4-14) or by  $S_y$  given by

$$S_y = \left[ \frac{1}{N} \sum_{k=1}^W y^2(K) \right]^{1/2} \quad (6-1)$$

The resulting autocorrelation was calculated from

$$R_y(\lambda) = \frac{\frac{1}{N} \sum_{k=1}^W y(K) y(K+\lambda)}{S_y^2}$$

For a time increment of  $T = 0.125$  and 1,000 samples the result in Figure 2-1 was obtained. This result is in good agreement with the desired autocorrelation. The value obtained for  $S_y^2$  for  $T = 0.125$  was 0.142 which is in good agreement with the result obtained theoretically which is given in Section IV as 0.145.

## **Section VII**

### **CONCLUSIONS**

A technique is developed for the simulation of random wind signals having an appropriate autocorrelation which can be readily generated on a digital computer. These results are to be used in generating wind data tapes for Space Shuttle launch simulations. These results can be linearly interpolated to give intermediate values between the generated results.

## **Section VIII**

### **REFERENCES**

1. Fichtl, G. H., Perlmutter, M., Simulation of Random Winds for Space Shuttle Ascent Control, NASA TN, to be published.
2. Dorf, R. C., Modern Control Systems, Addison-Wesley Co., Inc., 1967.
3. Ogata, K., Modern Control Engineering, Prentice Hall, 1970, New York.
4. Freeman, H., Discrete Time Systems, John Wiley and Sons, 1965.
5. Neuman, F. and Foster, J. D., Investigation of a Digital Automatic Aircraft Landing System in Turbulence, NASA TND6066, October 1970.
6. Laning, J. H. and Battin, R. H., Random Process in Automatic Control, McGraw Hill, 1956, New York.